Articulation Joint Drive Performance Envelope

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1 Basics

This technical specification outlines the enhanced drive model implementation for PhysX articulation joints. The goal is to improve sim-to-real transfer by more accurately capturing the drive's behavior. To achieve that, PhysX implements a static drive model which effectively approximates a set of achievable operating points in the (jointVelocity, driveEffort) plane with a piecewise linear function known as the performance envelope. It is important to keep in mind, that although it offers a fast, efficient and simple solution, it has some limitations: it does not capture dynamic effects, temperature dependencies, or high-frequency behavior.

The performance envelope is defined by two coupled constraints operating in the (jointVelocity, driveEffort) space. For rotational joints, these are expressed as:

$$|\tau| \leq \tau_{\max} - k_{\tau v} \cdot |v| \tag{1}$$

$$|v| \leq v_{\max} - k_{v\tau} \cdot |\tau| \tag{2}$$

Equation 1 defines the torque-speed boundary, limiting the maximum achievable torque τ at any given velocity v. At $v = 0$, the torque is capped at $\tau_{\max}$, with torque capacity decreasing linearly as velocity increases.

Equation 2 defines the speed-torque boundary, capping the maximum velocity v at any applied torque τ. At $\tau = 0$, the velocity reaches $v_{\max}$, with speed capacity decreasing linearly as torque demand increases.

The parameters that define these constraints are listed in Table 1, and are configurable through the drive model API.

Table 1: Drive Model Parameters

Parameter	Symbol	Units
maxEffort	$\tau_{\max}$	Nm
velocityDependentResistance	$k_{\tau v}$	Nm/(rad/s)
maxActuatorVelocity	$v_{\max}$	rad/s
speedEffortGradient	$k_{v\tau}$	(rad/s)/Nm

2 API

Drive performance envelope parameters are stored in a struct that accepts four parameters in the constructor:

```
PxPerformanceEnvelope(const PxReal maxEffort_,
                      const PxReal maxActuatorVelocity_,
                      const PxReal velocityDependentResistance_,
                      const PxReal speedEffortGradient_)
```

The performance envelope parameters may be specified using PxPerformanceEnvelope and passed to the PxArticulationDrive constructor:

```
PxArticulationDrive(const PxReal stiffness_, const PxReal damping_,
     const PxPerformanceEnvelope envelope_,
     PxArticulationDriveType::Enum driveType_ = PxArticulationDriveType::eFORCE)
```

3 Implementation

First, the drive impulse is computed as described in the accompanying *ImplicitDrives* document. Second, a two-stage clamping process (illustrated in Figures 1 and 2.) is applied to the total drive impulse denoted as λ_{drive} in this document. The total drive impulse consists of two components: the accumulated drive impulse from the implicit PD controller, and an external user-defined joint force applied at the beginning of the simulation step.

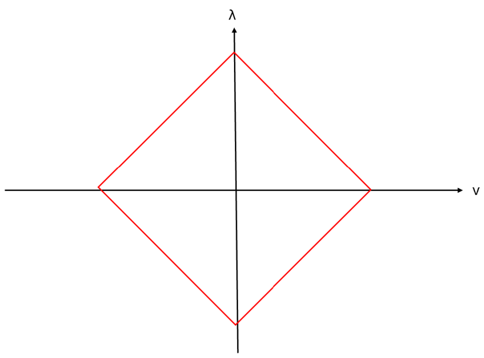

Figure 1: Enforcement of the velocity constraint according to Equation 2

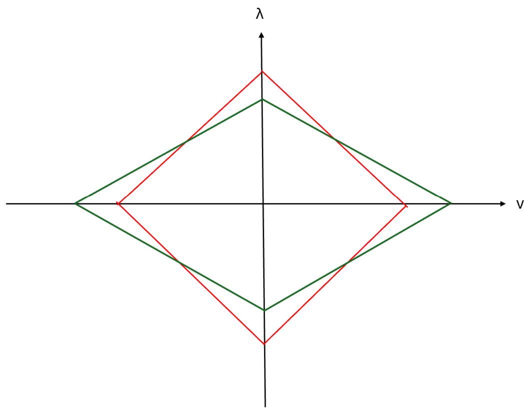

Figure 2: Enforcement of the effort constraint according to Equation 1

3.1 Velocity Constraint Enforcement

The joint velocity is governed by the following relationship:

$$v = v_0 + (\lambda - \lambda_0) \cdot r \tag{3}$$

where:

- v_0: joint velocity after λ_0 has been applied.
- λ_0: Pre-accumulated total drive impulse which includes the accumulated drive impulse from the implicit PD controller and external user-defined joint impulse applied at the beginning of the iteration.
- λ: New accumulated impulse that we wish to compute
- r: System response factor (positive constant)

3.1.1 Constraint Formulation

The velocity magnitude obeys the following inequality:

$$|v| \leq v_{\max} - k_{v\tau}|\lambda| \tag{4}$$

Decomposing the absolute value constraint:

$$-(v_{\max} - k_{v\tau}|\lambda|) \leq v \leq v_{\max} - k_{v\tau}|\lambda| \tag{5}$$

Substituting (3): into (5) reveals:

$$-(v_{\max} - k_{v\tau}|\lambda|) \leq v_0 + (\lambda - \lambda_0)r \leq v_{\max} - k_{v\tau}|\lambda| \tag{6}$$

3.1.2 Case Analysis for $\lambda > 0$

For positive impulse values ($\lambda > 0$), equation (6) becomes:

$$\begin{cases} \lambda(r - k_{v\tau}) \geq \lambda_0 r - v_0 - v_{\max} \\ \lambda(r + k_{v\tau}) \leq \lambda_0 r - v_0 + v_{\max} \end{cases} \tag{7}$$

Defining critical thresholds:

$$\beta_1 = \frac{\lambda_0 r - v_0 - v_{\max}}{r - k_{v\tau}} \tag{8}$$

$$\beta_2 = \frac{\lambda_0 r - v_0 + v_{\max}}{r + k_{v\tau}} \tag{9}$$

The system behavior depends on the sign of $r - k_{v\tau}$:

Case	**Condition**	**Feasible Region**
I-a	$r > k_{v\tau}$	$\lambda \in [\beta_1, \beta_2]$
I-b	$r < k_{v\tau}$	$\lambda \in (-\infty, \min(\beta_1, \beta_2)]$

Imposing the constraints $\lambda \geq 0$ and $\lambda \leq \lambda_{\text{drive}}$ yields the final set of solution cases:

Case	**Condition**	**Feasible Region**
I-a	$r > k_{v\tau}$	$\lambda \in [\max(0, \beta_1), \min(\lambda_{\text{drive}}, \beta_2)]$
I-b	$r < k_{v\tau}$	$\lambda \in [0, \min(\lambda_{\text{drive}}, \beta_1, \beta_2)]$

3.1.3 Case Analysis for $\lambda < 0$

$$\begin{cases} \lambda(r + k_{v\tau}) \geq \lambda_0 r - v_0 - v_{\max} \\ \lambda(r - k_{v\tau}) \leq \lambda_0 r - v_0 + v_{\max} \end{cases} \tag{10}$$

Case	**Condition**	**Feasible Region**
II-a	$r > k_{v\tau}$	$\lambda \in [\beta_1, \beta_2]$
II-b	$r < k_{v\tau}$	$\lambda \in [\max(\beta_1, \beta_2), \infty)$

where:

$$\beta_1 = \frac{\lambda_0 r - v_0 - v_{\max}}{r + k_{v\tau}}$$
$$\beta_2 = \frac{\lambda_0 r - v_0 + v_{\max}}{r - k_{v\tau}}$$

Imposing the constraints $\lambda \leq 0$ and $\lambda \geq \lambda_{\text{drive}}$ yields the final feasible solution space:

Case	**Condition**	**Feasible Region**
II-a	$r > k_{v\tau}$	$\lambda \in [\max(\lambda_{\text{drive}}, \beta_1), \min(0, \beta_2)]$
II-b	$r < k_{v\tau}$	$\lambda \in [\max(\lambda_{\text{drive}}, \beta_1, \beta_2), 0]$

3.2 Effort Constraint Formulation

The velocity relationship is defined in (3). The constraint $|\lambda| \leq \lambda_{\max} - k_{\tau v}|v|$ can be reformulated as $|v| \leq \lambda_{\max}/k_{\tau v}|v| - 1/k_{\tau v}|\lambda|$. This has the same form as the velocity constraint and we can formulate the solution analogously.

E.g. the solution for $\lambda > 0$:

Table 2: Constraint Solution Cases

Case	Conditions	Solution Bounds
1	$r + 1/k_{\tau v} > 0 \land r - 1/k_{\tau v} > 0$	$[\max(0, \beta_1), \min(\lambda_{\text{drive}}, \beta_2)]$
2	$r + 1/k_{\tau v} > 0 \land r - 1/k_{\tau v} < 0$	$[0, \min(\lambda_{\text{drive}}, \beta_1, \beta_2)]$

where:

$$\beta_1 = \frac{\lambda_0 r - v_0 - \lambda_{\max}/k_{\tau v}}{r - 1/k_{\tau v}}$$
$$\beta_2 = \frac{\lambda_0 r - v_0 + \lambda_{\max}/k_{\tau v}}{r + 1/k_{\tau v}}$$

After finding the lower and upper bounds for both effort and velocity constraint, the intersection of solutions is found and is used to clamp the impulse.

4 Datasheet-to-Sim

This section provides a guide for deriving the performance envelope parameters from a typical motor datasheet. While manufacturers may use varying units (e.g., rpm for speed, mNm for torque), the resulting parameters must be converted to SI units (rad/s, Nm) for compatibility with the PhysX API (see Table 1).

4.1 maxEffort

The maximum torque or force that an actuated joint can exert, denoted as $\tau_{\max}$, is often constrained by thermal limits of the motor and its components. Typically, this maximum effort is highest at low speeds, since motor losses tend to increase with velocity.

$\tau_{\max}$ can be determined by the most restrictive of the following factors:

- **Controller Current Limit:**

$$\tau_{\max} = I_{C,\max} \cdot k_t \cdot i_G \cdot \eta_G$$

  where:

  - $I_{C,\max}$: Maximum current the controller can deliver
  - k_t: Motor torque constant
  - i_G: Gear ratio
  - η_G: Gear efficiency

- **Motor Torque Limit:**

$$\tau_{\max} = \tau_{M,\max} \cdot i_G \cdot \eta_G$$

  where:

  - $\tau_{M,\max}$: The maximum torque the motor can deliver, which depends on the duration of torque application.

- **Gear Torque Limit:**

$$\tau_{\max} = \tau_{G,\max}$$

  where $\tau_{G,\max}$ is the maximum torque capacity of the gear itself.

4.2 velocityDependentResistance

The parameter $k_{\tau v}$ represents the slope of the torque limit as a function of velocity, characterizing the speed-dependent losses in the system. These losses primarily arise from iron losses in the motor core and viscous friction within the motor and gearbox.

Typically, $k_{\tau v}$ is not directly specified in datasheets. Instead, it can be inferred implicitly by analyzing the motor's operation diagram or performance curves, which illustrate how the maximum torque decreases with increasing speed due to these losses.

4.3 maxActuatorVelocity

The maximum achievable joint velocity ($v_{\max}$) can be limited by electrical or mechanical constraints. The principal limiting factors are:

- **Voltage Limit:**

$$v_{\max} = \frac{k_v \cdot V_{\max} \cdot \mu_C}{i_G}$$

  where:

  - $V_{\max}$: Maximum available supply voltage
  - k_v: Motor speed constant
  - μ_C: Controller modulation factor (usually ≈ 0.9 for PWM duty cycle limits)
  - i_G: Gear ratio

- **Motor Mechanical Limit:**

$$v_{\max} = \frac{n_{M,\max}}{i_G}$$

  where:

  - $n_{M,\max}$: Maximum rotational speed of the motor (mechanical limit)

- **Gear Input Speed Limit:**

$$v_{\max} = \frac{n_{G,\text{in},\max}}{i_G}$$

  where:

  - $n_{G,\text{in},\max}$: Maximum input speed rating of the gear

In practice, $v_{\max}$ is determined by the most restrictive of these factors under current operating conditions.

Note: While controllers theoretically have speed limits dependent on motor pole pairs, these are excluded from consideration here for simplicity. The presented model focuses on dominant electromechanical constraints.

4.4 speedEffortGradient

The parameter $k_{v\tau}$ represents the speed-torque gradient of the system, defined as the ratio of speed change to torque change ($\Delta v / \Delta \tau$). Its value depends on the dominant limiting factor in the drive system:

- **Voltage-Limited Case:**
  When the maximum speed is constrained by the motor's no-load speed, $k_{v\tau}$ corresponds to the motor's intrinsic speed-torque gradient:

$$\frac{\Delta n_{\text{mot}}}{\Delta T_{\text{mot}}} = \frac{R_{\text{mot}}}{k_t^2}$$

  where:

  - R_{mot}: Motor winding resistance
  - k_t: Motor torque constant

For motors with significant inductance (common in high-torque-density robotics applications), use:

$$\frac{\Delta n_{\text{mot}}}{\Delta T_{\text{mot}}} \approx \frac{n_0 - n_N}{\tau_N}$$

where:

- n_0: No-load speed
- n_N: Nominal speed
- τ_N: Nominal torque

The system-level gradient is scaled by the gear ratio i_G:

$$k_{v\tau} = \frac{\Delta n_{\text{sys}}}{\Delta T_{\text{sys}}} = \frac{\Delta n_{\text{mot}}}{\Delta T_{\text{mot}}} \cdot \frac{1}{i_G^2}$$

- **Mechanical-Limited Case:**
  If the maximum speed is restricted by mechanical constraints (motor/gear limits), then $k_{v\tau} = 0$, as velocity becomes torque-independent at saturation.