Tendon Dynamics

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Fixed Tendons

Introduction

Fixed tendons are an abstract mechanism that couple degrees of freedom (DOF) of an articulation. A fixed tendon is composed of a tree of tendon joints, where each joint is associated with exactly one axis of a link's incoming articulation joint. In the following, when we refer to a tendon joint's position, we mean the position of the axis of the associated articulation joint.

Each tendon joint has a coefficient that determines the contribution of the (rotational or translational) joint position to the length of the tendon, which is evaluated recursively by traversing the tree: the length at a given tendon joint is the length at its parent tendon joint plus its joint position scaled by the coefficient. The tendon joint coefficients may similarly be used to compute the speed of the tendon by considering the joint speeds instead of the joint positions.

Tendons may be configured so that the tendon length achieves a specified rest length and/or remains within a specified range. This is achieved by applying forces to each joint referenced by the tendon. Conceptually, the tendon applies two forces to each joint: a force that drives the tendon length towards the rest length of the tendon and a force to ensure that the tendon length stays within its lower and upper bounds.

Fixed tendons cannot link arbitrary DOFs of the articulation but must follow the articulation topology: a parent and child tendon joint must be associated with articulation joints that are connected by a single articulation link. This rule is imposed for the purposes of compute efficiency. Branching along the articulation topology is possible, as shown in Figure (1). As a final note, it is not permitted to associate a tendon joint with a fixed joint of the articulation.

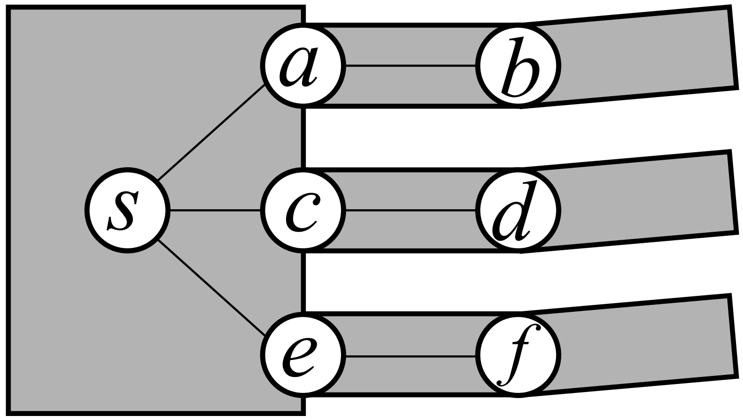

Figure 1: Tendons support linear chains of joints and branching.

Parameters

A PhysX fixed tendon is characterized by the following parameters.

R	Rest length
Θ	Length offset
c	Joint coefficient that scales joint position
r	Joint coefficient that scales the tendon force
γ	Spring stiffness
δ	Joint velocity damping
γ_{lim}	Limit stiffness
$\overline{L}$	Upper limit
$\underline{L}$	Lower limit

Dynamics

The length (L) and speed (S) of the tendon L have the following form

$$L = \Theta + \sum_i c_i \times p_i \tag{1}$$

$$S = \sum_i c_i \times \dot{p}_i \tag{2}$$

with p_i denoting the joint position of the ith joint in the tendon and $\dot{p}_i$ denoting the joint speed of the ith joint in the tendon.

The rest length is enforced by conceptually applying a force f_i^R to each joint of the tendon

$$f_i^R = (\gamma \times (R - L) - \delta \times S) \times r_i \tag{3}$$

In the event that the tendon length breaches the lower or upper limit a secondary force f_i^{Lim} is applied to each joint of the tendon

$$f_i^{Lim} = (\gamma_{lim} \times \Delta) \times r_i \tag{4}$$

with Δ denoting the change in tendon length required to bring the tendon length back to the nearest limit.

$$\Delta = \begin{Bmatrix} (\overline{L} - L) & \text{if } L > \overline{L} \\ (\underline{L} - L) & \text{if } L < \underline{L} \\ 0 & \text{otherwise.} \end{Bmatrix} \tag{5}$$

Note that PhysX implicitly integrates the tendon constraints; that is, the constraint forces (3) and (4) are computed at the end of the timestep, under the assumption that they are applied as a constant force during the time step. As a consequence, the user may choose large values for stiffness γ and limit stiffness γ_{lim} without hitting instabilities (suitably tuned damping will help with oscillations, however).

Spatial Tendons

Introduction

Spatial tendons create line-of-sight distance constraints between links of a single articulation. In particular, spatial tendons run through attachments that are positioned relative to an articulation link, and their length is defined as a weighted sum of the distance between the attachments in the tendon. It is possible to create multiple attachments per link, for example for tendon-routing purposes. In contrast to fixed tendons, spatial tendons are not constrained to follow the articulation topology.

Similar to fixed tendons, spatial tendons may branch, in which case the tendon splits up into multiple conceptual sub-tendons, one for each root-to-leaf path in the tendon tree. Length and limit constraints are evaluated per sub-tendon, and have spring-damper dynamics that may both contract and extend the tendon (one may use appropriately set limits to achieve a one-sided, string-like constraint).

Each sub-tendon constraint force acts on the corresponding leaf and root links.

Parameters

A PhysX spatial tendon is characterized by the following parameters.

R	Rest length - defined per leaf
Θ	Length offset - applies to all sub-tendons
c	Coefficient that scales distance between an attachment and its parent
γ	Spring stiffness - applies to all sub-tendons
δ	Tendon-length damping - applies to all sub-tendons
γ_{lim}	Limit stiffness - applies to all sub-tendons
$\overline{L}$	Upper limit - defined per leaf
$\underline{L}$	Lower limit - defined per leaf

Dynamics

Let the subscript s refer to a leaf attachment and the sub-tendon that it defines. The length of the sub-tendon from leaf s to the root is

$$L_s = \Theta + \sum_i c_i \times |x_i - x_{i-1}| \tag{6}$$

where x_i and x_{i-1} refer to the world position of the i-th tendon attachment and its parent, respectively. The iterator variable i runs through the sub-tendon from leaf to root (excluding the root, to be precise).

Each sub-tree has an associated velocity S_s

$$S_s = \frac{d}{dt}\left(|x_s - x_{s-1}| + |x_{1,s} - x_0|\right) \tag{7}$$

where x_{s-1} is the world position of the leaf's parent attachment, x_0 is the position of the root, and $x_{1,s}$ is the position of the root's child that leads to the leaf (hence the additional s subscript). Note that the length-scaling coefficients do not factor into the damping force, nor are the tendon velocities of any further intermediate segments considered.

Similar to fixed tendons, spatial tendons apply a force that drives the sub-tree length towards the rest length of the tendon and a force to ensure that the sub-tree length stays within its lower and upper bounds. The scalar constraint force of the sub-tendon is

$$f_s = \gamma \times (R_s - L_s) + \gamma_{lim} \times \Delta_s - \delta \times S_s. \tag{8}$$

with denoting the change in sub-tree length required to bring the length back to the nearest limit

$$\Delta_s = \left\{\begin{matrix} (\overline{L}_s - L_s) & \text{if } L_s > \overline{L}_s \\ (\underline{L}_s - L_s) & \text{if } L_s < \underline{L}_s \\ 0 & \text{otherwise.} \end{matrix}\right\} \tag{9}$$

As with fixed tendons, PhysX implicitly integrates spatial tendon constraints.